The x and y displacements are transformed successively into velocity and then acceleration demands and fed into Earth axis acceleration response control laws to achieve the (x, y) position.

V/STOL Aircraft Responses

The translational motion control laws are applied to a V/STOL aircraft using off-line simulation. The NDI forces and moments are injected directly into the aircraft equations to achieve the prescribed response. The aircraft is required to attain heave acceleration $a_{Ez\,\mathrm{dem}} = 0.1\,g$ and forward velocity $v_{Ex\,\mathrm{dem}} = 60$ ft/s via two-dimensional thrust vectoring in the x–z plane and lateral translation rate $v_{Ey\,\mathrm{dem}} = -10$ ft/s via bank angle from a 32-ft/s trim condition. Figure 1 shows the time responses of the aircraft to these commands after 2 s of open-loop flight from trim. The responses show accurate achievement of the commands within 4–8 s. Some form of attitude control must be applied to prevent the aircraft rolling, pitching, and yawing during translation maneuvers. In this example, pitch attitude, calculated using Eq. (6) and implemented as described in Ref. 5, is set to be compatible with commanded translation to prevent the aircraft pitching.

These commands are specifically chosen to demonstrate the ability to mix different response types in the three body-referenced Earth axes. Response mixing requires NDI forces and moments to work together compatibly. With velocity and attitude/position commands, the authority of forces and moments can be controlled by the choice of bandwidths. The bandwidths limit accelerations and velocities and hence serve as a means of governing the degree of control exercised by each response type.

Conclusions

This Note has presented and demonstrated nonlinear dynamic inversion algorithms for translation motion control of V/STOL aircraft using three-dimensional and two-dimensional thrust vectoring. The use of the exact nonlinear dynamic inversion approach as a fast prototyping tool in control law design has been proposed. Further research is examining the effects of actuation, sensors, discrete digital elements, and structural dynamics to develop a very complete approach to aid engineers in the development of appropriate control laws and vehicle configurations.

To translate conceptual control into real or actual control, the dynamic inversion forces and moments must be mapped into demands on engine and aerodynamic control surfaces. The development of such methods with due regard to motivator redundancy, blending between hover and forward flight, robustness to gusts and aerodynamic coefficient uncertainties, and actuator limits continues to receive much research attention. The new algorithms have a useful role in resolving the first two of these issues because they make use of both aerodynamic and thrust vector control to achieve a single response.

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Mixed μ-Synthesis via H_2-Based Loop-Shaping Iteration

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I. Introduction

ENGINEERS encounter the robust control problems with respect to structured mixed (real and complex) uncertainty in many practical applications. Such a concept of mixed uncertainty provides control engineers with a more proper description and less conservative design than the general uncertain systems. Unfortunately, the computation of structure singular value with respect to mixed perturbation sets is very complex and known to be nonpolynomial (NP) hard.[1] As a result, the mixed μ-synthesis becomes a more challenging problem.

There are two representative approaches, D, G-K iteration and μ-K iteration,[2,3] to mixed μ-synthesis. Although a few successful designs have been presented, the practical use of D, G-K iteration is obstructed by the need for fitting purely complex scalings with high-order, all-pass transfer functions, as well as the need for fitting in phase and in magnitude. In comparison with D, G-K iteration, the advantage of μ-K iteration is that one needs to fit in magnitude only; however, the disadvantage is that one needs to compute not only the mixed but also the corresponding complex μ upper bounds at each iteration. Therefore, the μ-K iteration still is quite involved and computationally demanding.

A new approach, called the H_2-based loop-shaping iteration, has been proposed and successfully applied to H_∞-synthesis and the complex μ-synthesis.[4,5] This H_2-based mixed μ-synthesis approach is totally different from the H_∞-based mixed μ-synthesis approaches (D, G-K iteration and μ-K iteration). The mixed μ-controller is synthesized by performing a sequence of weighted H_2 optimizations, not H_∞ optimizations; therefore, this approach is less computationally demanding. In addition, the algorithm is quite easy to implement.

II. Reviews on μ

Consider a generalized augmented plant P partitioned as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{1}$$

To test for robust performance, we first pull out the uncertain perturbations and rearrange the uncertain system into an $N\Delta$-structure. N is the lower linear fractional transformation of augmented plant P closed by controller K, and M is the upper linear fractional transformation of N closed by uncertainty Δ:

$$\begin{aligned} N &= F_l(P, K) \\ &= P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \end{aligned} \tag{2}$$

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$$M = F_u(N, \Delta)$$
$$= N_{22} + N_{21}\Delta(I - N_{11}\Delta)^{-1}N_{12} \quad (3)$$

The uncertainty Δ is a member of the bounded subset $B\Delta$:

$$B\Delta = \{\Delta \in \Delta_s | \bar{\sigma}(\Delta) < 1\} \quad (4)$$

where $\bar{\sigma}$ denotes the largest singular value and Δ_s represents the structure perturbation set

$$\Delta_s = \Big\{\text{diag}\big(\delta_1^r I_{r_1}, \ldots, \delta_{m_r}^r I_{r_{m_r}}, \delta_1^c I_{r_{m_r+1}}, \ldots, \delta_{m_c}^c I_{r_{m_r+m_c}},$$
$$\Delta_1, \ldots, \Delta_n\big) \Big| \delta_i^r \in R, \delta_i^c \in C, \Delta_j \in C^{r_{m_r+m_c+j} \times r_{m_r+m_c+j}}\Big\} \quad (5)$$

The structured singular value μ of N with respect to Δ is defined as

$$\mu_\Delta(N) \equiv \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in B\Delta, \det(I - N\Delta) = 0\}} \quad (6)$$

Give an H_∞ norm performance specification

$$\|F_u(N, \Delta)\|_\infty < 1 \quad (7)$$

Rearrange the uncertain system into an $N\Delta$-structure by introducing a fictitious uncertainty block Δ_P, which itself is a full complex matrix stemming from the H_∞-norm performance specification. Then, the closed-loop system is performance robust, i.e., $\|F_u(N, \Delta)\|_\infty < 1$ for all $\Delta \in B\Delta$ iff

$$\|\mu_{\tilde{\Delta}}(N)\|_\infty \leq 1 \quad (8)$$

where the enlarged perturbation set $\tilde{\Delta}$ is

$$\tilde{\Delta} = \big\{\text{diag}(\Delta, \Delta_P) | \Delta \in B\Delta, \Delta_P \in C^{k\times k}, \bar{\sigma}(\Delta_P) < 1\big\} \quad (9)$$

III. Algorithm of H_2-Based Loop-Shaping Iteration

The procedure of H_2-based loop-shaping iteration for mixed μ-synthesis is summarized as follows.

Initialization: $i = 0$

1) Set $\hat{W}_0(s) = 1$.

2) Compute $K_0 = \arg\inf_K \|\hat{W}_0 F_l(P, K)\|_2$ using the H_2 optimization technique.

3) Compute $\gamma_0(w) = \mu_{\tilde{\Delta}}[\hat{W}_0 F_l(P, K_0)(j\omega)]$.

4) Compute $\lambda_0 = \|\hat{W}_0 F_l(P, K_0)\|_2$.

Recursive formula: $i = 1, 2, 3, \ldots$

1) Set $\deg(\hat{W}_i) = n_w$, where $\hat{W}_i(s)$ is a scalar, minimum-phase transfer function.

2) Fit $\gamma_{i-1}(\omega)/\lambda_{i-1}$ by $|\hat{W}_i(j\omega)|$.

3) Compute $K_i = \arg\inf_K \|\hat{W}_i F_l(P, K)\|_2$.

4) Compute $\gamma_i(\omega) = \mu_{\tilde{\Delta}}[\hat{W}_i F_l(P, K_i)(j\omega)]$.

5) Compute $\lambda_i = \|\hat{W}_i F_l(P, K_i)\|_2$.

6) Repeat the recursive formula until no further reduction in $\|\mu_{\tilde{\Delta}}[F_l(P, K_i)(j\omega)]\|_\infty$ can be achieved.

Practically, the structured singular value $\mu_{\tilde{\Delta}}[F_l(P, K_i)(j\omega)]$ may become flat, and $\|\mu_{\tilde{\Delta}}[F_l(P, K_i)(j\omega)]\|_\infty$ approximates to a constant for a wide frequency range after a few iterations. The extent of flatness depends heavily on the accuracy of calculating $\mu_{\tilde{\Delta}}[\hat{W}_{i-1}F_l(P, K_{i-1})(j\omega)]$ and curve fitting $\gamma_{i-1}(\omega)/\lambda_{i-1}$ by $|\hat{W}_i(j\omega)|$.

Remark. The H_2-based loop-shaping iteration depends heavily on good curve fitting in recursive formula (1) and (2), preferably by a transfer function $\hat{W}_i(s)$ of lower order. The main reason for preferring a low-order fitting is that it reduces the order of the H_2 problem, which usually improves the numerical properties of the corresponding H_2 optimization and also yields a controller of lower order. However, this way will probably cause a slow iteration convergence.

IV. Noncollocated Satellite Attitude Control

Consider a satellite, which is suggested by the need to control the pointing direction, or attitude, in orbit about the Earth. Physical analysis leads us to assume that there are parameter variations as a result of temperature fluctuation. Moreover, to measure the attitude angle of the vehicle, the sensor and actuator are noncollocated. This makes the system much harder to control. The objective of this problem is to maintain an acceptable performance in the presence of parameter variations. The satellite structure is modeled as a simple spring–mass–damping system, where the spring constant k and damping constant b are assumed to vary in certain ranges.[6] However, this simplified system has crucial features of many more complex space structures.

The equations of motion are derived as

$$J_1\ddot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = T_c \quad (10a)$$

$$J_2\ddot{\theta}_2 + b(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0 \quad (10b)$$

$$0.09 \leq k \leq 0.4 \quad (11)$$

$$0.0036 \leq b \leq 0.04 \quad (12)$$

where T_c is the control torque and θ_2 is the output to be controlled. With inertias $J_1 = 1$ and $J_2 = 0.1$, the transfer function from T_c to θ_2 is

$$G(s) = \frac{10bs + 10k}{s^2(s^2 + 11bs + 11k)} \quad (13)$$

The pointing requirement of a satellite arises when it is necessary to point the unit from one direction to another direction. Explicitly, this can be specified by the dynamics of the step response with settling time t_s less than 20 s and overshoot M_p no more than 15%. According to the performance specifications in the time domain, we select the performance weighting $W_p(s)$ by shaping complementary sensitivity function

$$W_p(s) = \frac{0.85(s^2 + 0.696s + 0.36)}{(s + 0.696)(s + 0.0001)(0.0001s + 1)} \quad (14)$$

After performing the H_2-based loop-shaping iteration eight times (Table 1), we obtain a mixed μ controller with 14 states. The magnitude plot of $\mu_{\tilde{\Delta}}[F_l(P, K_i)(j\omega)]$ is shown in Fig. 1. It can be recognized easily that the final curve is approximately flat and is actually less than one, indicating that the robust performance requirement certainly is satisfied. Finally, as shown in Fig. 2, the performance robustness achieved by the resulting mixed μ controller can be verified further by the step responses of 121 perturbed plants, where the parameters k and b are selected as

$$k = \{0.245 + 0.155\delta_k \,|\, \delta_k \in [\pm 1, \pm 0.8, \pm 0.6, \pm 0.4, \pm 0.2, 0]\} \quad (15)$$

$$b = \{0.0218 + 0.0182\delta_b \,|\, \delta_b \in [\pm 1, \pm 0.8, \pm 0.6, \pm 0.4, \pm 0.2, 0]\} \quad (16)$$

We make the following remarks on the satellite example.

1) In all approaches to μ-synthesis (including mixed μ-synthesis), a resulting high-order controller is almost inevitable. By applying

Table 1 Iteration results of satellite's attitude control

Iteration no. i	$\|\mu_{\tilde{\Delta}}[F_l(P, K_i)(j\omega)]\|_\infty$	$\|\hat{W}_i F_l(P, K_i)\|_2$
0	2.5646	62.6941
1	1.6297	0.9103
2	1.3212	0.9279
3	1.1756	0.9271
4	1.0853	0.9181
5	1.1109	0.8975
6	1.0311	0.8923
7	1.1201	0.8863
8	0.9715	0.8820

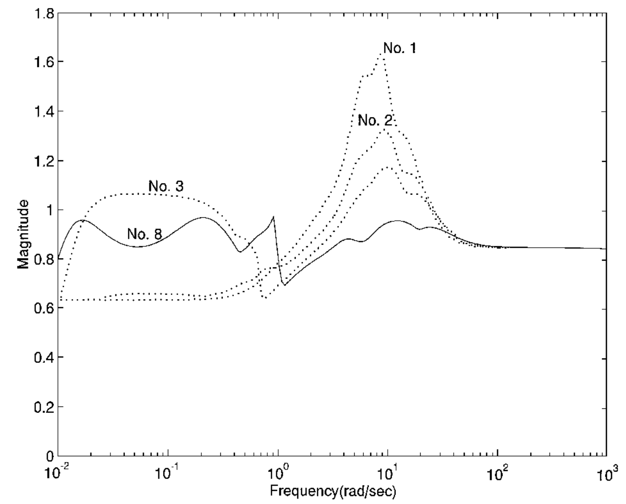

Fig. 1 Upper bounds of $\mu_{\hat{\Delta}}[F_l(P, K_i)(j\omega)]$ for iterations 1, 2, 3, and 8.

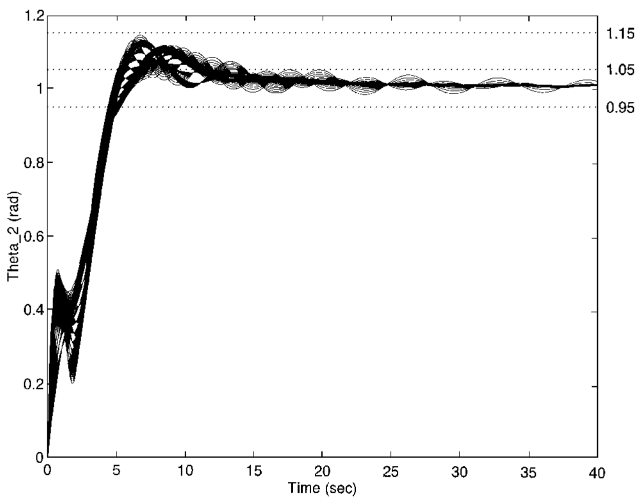

Fig. 2 Step responses of 121 perturbed satellite plants with mixed μ controller.

model reduction methods, it is possible to reduce the controller order to a lower level with acceptable degradation in control performance. However, using controller approximation with fewer states implies an increase in μ. Still, there are open numerical problems in connection with actual implementation in μ-synthesis.

2) The robust performance (RP) condition in Eq. (8) is identical to a robust stability (RS) condition with an additional perturbation block Δ_P. In other words, the RP condition is more severe than the RS condition, and so there is no doubt of RS in this illustrative satellite case, where the RP condition is satisfied.

V. Conclusions

We extend the H_2-based loop-shaping method to mixed μ-synthesis. A noncollocated satellite's attitude control design is formulated to the mixed μ-synthesis problem. The resulting controller with RP property from the μ criterion is obtained by a sequence of weighted H_2 optimizations. Also, the simulation results show that the time-domain performance specifications on settling time and overshoot are completely satisfied under real parameter variations. This successful design stresses the superior ability of the H_2-based loop-shaping method for mixed μ-synthesis.

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Linearization of a Six-Degree-of-Freedom Missile for Autopilot Analysis

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Introduction

TYPICALLY, the design of autopilots for missiles is based on a loop-at-a-time philosophy; that is, the lateral channels (yaw and pitch) are designed, and then the roll channel is designed separately.[1] Most of these autopilots have a fixed structure whose gains are then scheduled upon flight conditions such as altitude, Mach, and angle of attack. Gain selection is accomplished by examining both the performance and the relative stability of a linear representation of the missile at various points in the flight envelope. Most often, classical phase and gain margins are used as a measurement of relative stability. The worst-case values of these margins are obtained by breaking a single loop in a coupled lateral-roll model. Such design approaches are insufficient for obtaining autopilots that a priori account for robustness and performance. With the advances in control theory, it is now possible to design multivariable controllers that can account for both relative stability (robustness) and performance.[2] To perform such design and analysis, one must work with a fully coupled dynamic representation of the missile. This Note describes a way to properly linearize the dynamics for a fully coupled, six-degree-of-freedom (DOF), cruciform missile in trim. One accepted definition of trim for missiles that can attain large angles of attack is as follows: 1) the moments acting on the missile are zero and 2) the rate of change of both the angle of attack and the sideslip angle are zero. Note that this definition does not preclude nonzero roll rates at trim. An additional contribution of this Note is the derivation of a technique for determining the initial roll rates that are consistent with the dynamics of a trimmed missile as just defined.

Development of Linear Model

We define the following quantities for a missile with respect to Fig. 1: (x, y, z) are the missile body axes, (u, v, w) the projection of the missile velocity vector onto (x, y, z), and (p, q, r) the projection of the missile angular rate vector onto (x, y, z).

Note that (u, v, w) and (p, q, r) are, in fact, inertial values written with respect to the body axes. The analysis presented here is for a missile that is symmetric about its principal axes; thus, the inertial dyad is $\boldsymbol{I} = \mathrm{diag}([I_{xx} \;\; I_{zz} \;\; I_{zz}])$. We divide the forces and moments that act on the missile into two categories: 1) forces and moments due to missile aerodynamics including canards, wings, or fins; and

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